2.5

$$\#8 \quad \overset{M}{(3x^2+y)}\,dx + \overset{N}{(x^2y-x)}\,dy = 0$$

exact? $\dfrac{\partial M}{\partial y} = 1 \neq \dfrac{\partial N}{\partial x} = 2xy-1$ so Not exact.

consider $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \dfrac{1-(2xy-1)}{x(xy-1)} = \dfrac{2-2xy}{x(xy-1)} = \dfrac{-2(xy-1)}{x(xy-1)}$

$= \dfrac{-2}{x}$ ☺☺ a function of just $x \Rightarrow$

$$\mu(x) = e^{\int\left(\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}\right)dx} = e^{-2\int \frac{1}{x}dx} = e^{-2\ln x} = \frac{1}{x^2}$$

is an integrating factor, Then

$$\frac{1}{x^2}(3x^2+y)\,dx + \frac{1}{x^2}(x^2y-x)\,dy = 0$$

$$\overset{M}{(3+yx^{-2})}\,dx + \overset{N}{(y-x^{-1})}\,dy = 0$$

$\dfrac{\partial M}{\partial y} = x^{-2} = \dfrac{\partial N}{\partial x} = x^{-2}$ so now exact, solve as usual!

2.5

#10 $\overset{M}{(2y^2+2y+4x^2)}dx + \overset{N}{(2xy+x)}dy=0$

$\frac{\partial M}{\partial y} = 4y+2 \neq \frac{\partial N}{\partial x} = 2y+1$ Not exact

Consider $\dfrac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \dfrac{(2y+1)-(4y+2)}{2y^2+2y+4x^2} = \dfrac{-2y-1}{2y^2+2y+4x^2}$ NO HELP

Consider $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \dfrac{(4y+2)-(2y+1)}{x(2y+1)} = \dfrac{2y+1}{x(2y+1)} = \dfrac{1}{x}$ $\therefore$ a function of x.

Then $\mu(x) = e^{\int \frac{1}{x}dx} = x$ is an integrating factor

Then $\overset{M}{(2xy^2+2xy+4x^3)}dx + \overset{N}{(2x^2y+x^2)}dy=0$

$\frac{\partial M}{\partial y} = 4xy+2x = \frac{\partial N}{\partial x} = 4xy+2x$ $\therefore$ Now EXACT!